Bringing LP to Life

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The pressure on funding and the increasing size of classes in universities calls for different approaches to teaching. The increasing power of personal computers and the wide variety of software available today provides us with one opportunity for coping with these problems. This paper describes a system designed to help students learn the basics of Linear Programming (LP). The system uses animation to show how feasible regions are displayed (for two and three variable problems), which of the many (infinite) feasible solutions are of interest (the vertices), how to obtain the optimal solution, and how sensitive the solution is to changes in some of the data. The software complements rather than replaces the typical lecture or 'chalk and talk' presentation. The animations have been created in Animator Pro and 3D Studio from Autodesk. The paper will also describe how this system relates to other computer assisted learning software used in teaching LP, and discusses the importance of taking into a ccount the differing learning styles used by students.

1. Introduction

The pressure on funding and the increasing size of classes in universities calls for different approaches to teaching. Coping with large classes involves different teaching and learning accommodation, more learning resources such as books, printed materials and information technology, support staff and special technical support services. The overall balance between teaching staff and other resources will need to be different if relatively expensive teaching staff is to be employed to maximum effect. We need to encourage students to learn more independently of tutors and to this end we are introducing courses to 'teach' students to be independent. Students must help themselves and help each other. Students can teach each other, give seminars in groups without tutors, comment and mark each other's work, design each other's experiments, supervise themselves. The effect is to move decisively away from a dependence on contact hours as the prime management tool for teaching towards measures based on quality, innovation, and effective teaching. All of these will alter the balance of resources. The increasing power of personal computers and the wide variety of software available today also provides us with an opportunity for radically altering this balance. There is a need to introduce innovative approaches and structures, and to make the best of the new technology.

The Teaching and Learning Technology Programme of the Universities Funding Council in the UK (Darby, 1993) is a good example of an attempt to address this problem. "The aim of the programme is to make teaching and learning more productive and efficient by harnessing modern technology" (UFC, 1992). One of the 43 projects is MENTOR: Multimedia Educational Technology in Operational Research. The first phase of this project consists of five modules; introduction to OR, line ar programming (LP), simulation, stock control and forecasting. The LP module, like the others will be based in Windows (using a bespoke authoring software package) and run on a minimum 386 machine with VGA standard monitors, and 4Mb of memory, and will incorporate still images, video and some animation.

Animation software such as that briefly described later allows the less artistically inclined to generate relatively professional animated presentations. Animation is not new. Most of us have seen a Tom and Jerry cartoon! A local authority in the USA used animated computer graphics as a part of its annual budget presentation (Beagle, 1989). Animation sells well-known products such as Coca Cola, where 'Max Headroom', the British computer generated talk-show host is used in commercials (Spain, 1986). Some companies even prefer to use 'interactive personalities', computer-enhanced images of actors to make presentations to staff (Dobinski, 1989). Animation is a familiar tool in visual interactive simulation (Pidd, 1992). Several commercial packages, e.g. WITNESS (Al–Amin Hossain and Tobias, 1991), are available that allows one to animate the simulation of, say, a manufacturing system. In these systems you can see parts, tools and machines moving around the screen as the states change. Hurrion (1993) also suggests that "3D animation of the results of a simulation, if used in an interactive manner, can help in obtaining a much deeper understanding of the original system under investigation". Scientists have realised that they could make visible complex dynamic processes that once were invisible. "Perhaps large quantities of complex information can only be understood by people in any detail through the creation of 'pretty' pictures, sometimes lots of pictures — hence animation" (Dorling and Openshaw, 1992). And as La Breque (1989) put it "When I first saw the animation, I watched it over and over again. I thought that something was going on — but never exactly this".

Animation need not be expensive. The software used in this exercise cost several hundred pounds (even after a substantial educational discount), but this is a one off cost as the animations developed can be played back on many machines, more than fifty in this case, through the use of a free run-time module. A free player is also available to play back the animations in Windows. One system was developed "with software costing less than a round of beer, a cheap school computer, , and a little imagination" (Dorling and Openshaw, 1992).

The notion that animation can be a useful tool for teaching is becoming more and more widely accepted (Park and Gittelman, 1992). There are now several packages available that are used in conjunction with the more formal teaching approach; sorting (Brown, 1987), electronic circuit troubleshooting (Park and Gittelman, 1992), Newton's Laws (Reiber, 1990), visualising space-time patterns in geography (Dorling and Openshaw, 1992) and understanding molecular processes in chemistry (Buell et al, 1993). Park and Gittelman (1992) suggested that animation has three primary roles: as a device for attracting attention and maintaining motivation; as a means for representing domain knowledge involving explicit or implicit movement; and as an aid for explaining complex knowledge or phenomena.

The objective of the system described in this paper is to help students understand the concepts of LP by firstly attracting their attention, giving some feel for the type of problem that can be solved by LP, and introducing ideas on how these problems are solved. The tutorial presents two problems, the Barman's problem (devised by the author) and the Farmer's Problem (Littlechild, 1977). The software can not solve problems specified by the user. The animation enables the student to travel through, at his/her own pace, a sequence of menus and screen displays/animations that explain each stage (variables, objective function and constraints) in the construction of the LP model. The menus also allow the student to analyse what happens when parameters (objective coefficients and right hand sides) are changed by showing animations for these changes.

How this system matches the learning styles of students, relates to other software used in teaching, and will be extended in future work will also be discussed.

2. The Authoring Software

Animator Pro (AniPro) from Autodesk, is a 2D paint and animation software package that enables the user to create dynamic, high resolution (with the appropriate graphics card) presentations on a personal computer. It can turn "your VGA display into a silent alternative to Saturday morning TV" (Rosenberg, 1990). AniPro is essentially two programs. Its primary aim is to create moving images or images that change over time. It is also a sophisticated paint program for producing the individual 'frames' which wh en combined generate animations or 'flics' (Hartsham, 1992). AniPro has the typical paint program (such as Paintbrush) tools, draw, line, box, and circle, for drawing. The shapes drawn are not ordinary shapes but referred to as *tweenables*. Tweenables can easily be transformed to other tweenables by simply specifying the starting and ending tweenables, and the number of frames. AniPro's *polymorphic tweening* can transform "a frog into a handsome prince" (Rosenberg, 1990). AniPro comes with a wide range of ready made single frame pictures in the form of .GIF files, and animations as .CEL files. An animation is stored as a .FLC file, a .FLI file in earlier versions. Flics are joined using a 'script' file. A script file can be created not only to determine the order of the animation but also, with the help of menus, allows the user to select different parts of the animation. Supplied with AniPro is the Animator Pro Player program, ANIPLAY (AAPLAY in earlier versions) which is a flic 'projector' program that lets you play animations or display pictures, either one at a time or following a text script file. (Unfortunately the Windows version, AAWIN does not support the script files for ANIPLAY.)

3D Studio (3DS), also from Autodesk, is a complete, professional 3–D modelling and animation package for the personal computer. 3DS is essentially five programs in one; 2–D Shaper, 3–D Lofter, 3–D Editor, Materials Editor, and Keyframer. In the 2–D shaper you create the spline polygons, essentially the profiles of shapes used in other modules; for example a square. In the 3–D Lofter the spline polygons are converted into mesh objects, for example a cube from a square. This process is called lofting but one can think of it as a process of extrusion. However, the cross section need not be constant. The path of extrusion can be curved, objects can be twisted and even different shapes can be included in the path. A mesh object created in the 3D Lofter are transferred to the 3D Editor (although simple mesh objects such as cubes can be created in 3D Editor) along with many other mesh objects. The 3D Editor controls the positioning of objects, lights and camera. Lights come in three types — ambient, omni and spot — and are adjustable for colour and brightness. The view of the objects is determined by the camera position. A camera is given a target with lens focal length ranging from 15mm wide-angle to 200mm telephoto. Materials are assigned to objects. 3DS includes a library of over 100 materials but additional materials can created in the Materials Editor. You create animation in the Keyframer by altering, over a range of frames, the objects, cameras, and lights that have been arranged in the 3D Editor. There is also a sixth module, used in 3D Editor and Keyframer, called the Renderer. The Renderer analyses the mesh geometry of objects, lights and cameras, and applies the materials to the surface of the objects. This can be the slowest process in the creation of animations, sometimes taking many hours or even days to render all frames of an animation. The animations are stored as .FLC files. The .FLC files can be edited in AniPro and played back with ANIPLAY or AAPLAY. See Loveria (1992) for an extensive review of 3D Studio.

Models from AutoCad and other computer aided design packages can be taken into 3D Studio for visualisation and output to video. 3D Studio was chosen to create an animation of the proposed facilities in Manchester's bid for the Olympics in the year 2000, and used by a Formula One racing team to display the positions of logos to potential sponsors.

3. Teaching Linear Programming

Linear Programming is a well-known problem-solving technique (see Figure 1). LP has been applied to a wide range of problems; product mix, blending (fuel oils, fertilizers, animal foods), portfolio selection, advertising mix, distribution, etc. Typically the objective is the maximisation (profit, say) or minimisation (cost, say) of some linear function of the decision variables. The search for a solution is restricted by constraints, also linear with respect to the decision variables. Simple two variable problems can be described graphically. See Anderson et al (1988) for a good introduction to LP. The content of a course on the introduction to LP typically consists of the formulation of problems, description and solution of simple (two variables') problems using graphical techniques, the simplex method, solution of prob-

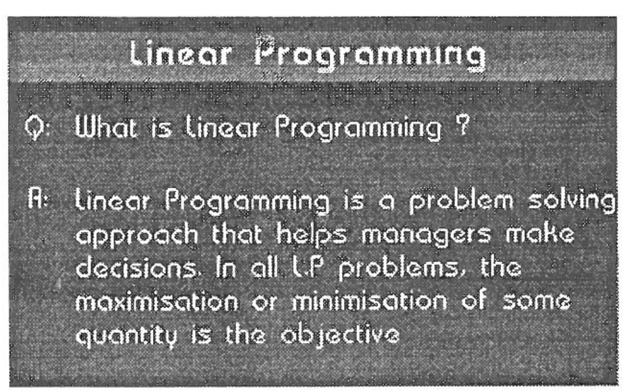

Fig. 1. What is Linear Programming?

lems with LP software, interpretation of solutions, and sensitivity analysis. A course will generally be a mix of lectures, tutorials, problems, case studies, and computer-based exercises. The balance between pedagogical methods will depend on the class size and resources available. In the author's case the class size is in excess of 150 and growing. The course consists of 14 lectures, 5 examples' classes and 10 open-door sessions. Several assignments, including computer-based work, are given as part of the continuous assessment.

When teaching LP in a class room setting one typically relies on 'chalk and talk' with some help from some overhead projector slides. One tries to 'animate' the process by drawing the constraints and the objective function, and moving lines (by redrawing) to illustrate the process. Overhead slides can also help by moving lines drawn on pieces of transparent film to reinforce this. This method is also useful in conveying what is happening in sensitivity analysis by pivoting the objective function (for objective ranging) and sliding constraints (for right hand side ranging). However, the student has difficulty in taking 'notes' on this process. It is difficult to describe what is happening with a pen and a piece of paper, the medium for taking notes in a lecture. The aim of the animation is for the student to repeat the animations in his/her own time to reinforce the process described in lectures. In the examples' classes, as the title suggests, more examples are described. The open door sessions are for one-to-one contact with students to sort out specific problems with the material.

Students will also use the LP module of the Mentor project and introduced to XPRESSMP for solving LP problems. The Mentor system is a Windows based icon driven system that includes all the areas covered in an introductory LP course and includes maps and histories of screens displayed. The LP module includes some self assessment exercises and has the potential for external assessment and monitoring. The authoring software used to develop the teaching modules is freely available to all teachers allo wing them to modify the modules to suit their own specific requirements. XPRESSMP is a comprehensive DOS based mathematical programming package. XPRESSMP is run in a DOS window in Windows and support staff have written an ASCII editor, similar to Notepad, that saves models in files with the required .MOD file extension. Large problems already formulated and stated as XPRESSMP files can be downloaded to students via the Novell network.

4. The Animation System

The animation begins with a 'fun' screen to suggest to students that they might enjoy what is to come (see Figure 2). (The mouse is a part of one of the 'CEL's, ready made animations supplied with AniPro)

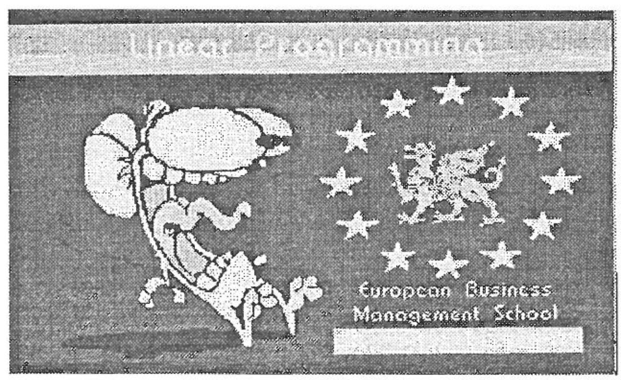

Fig. 2. Start-up Screen

The system currently contains two examples; the barman's problem and the farmer's problem. The description of the farmer's problem is shown in Figure 3. The user is shown how the written statement of the problem is translated into the LP formulation. The variables are defined, the constraints are constructed, and the objective function is determined. This is accomplished through several screens that the student can move through at his/her own pace, repeating this process if necessary before proceeding to

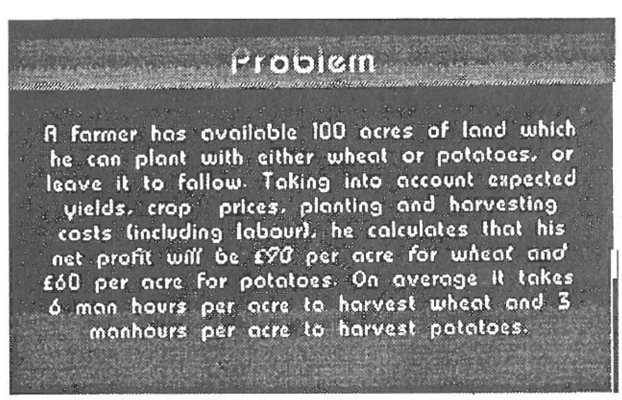

Fig. 3. The Farmer IPL's Problem

the next stage. Both problems contain only two variables and can be represented graphically. In representing these problems in this way it can be demonstrated that we are concerned with solutions on the boundary of the feasible region (see Figure 4). It is also demonstrated that of these solutions we need only be concerned with corners or vertices on the feasible region.

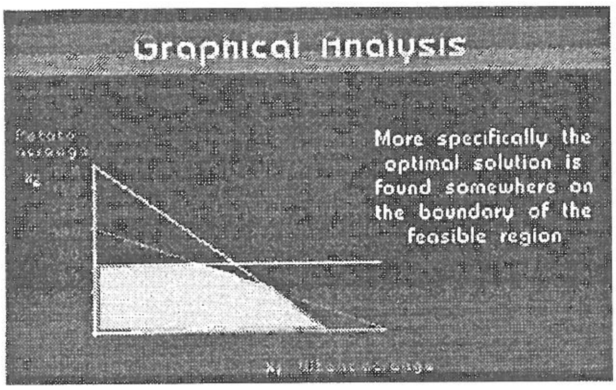

Fig. 4. Boundary of Feasible Region

Simple two variable problems can be solved by direct evaluation of all vertices or by sliding the objective function. Both methods are shown through simple animations. The last flic of the direct evaluation animation, with vertex on the 'move', is shown in Figure 5. After seeing the processes for determining the optimal solution, the student can then proceed to the section dealing with sensitivity analysis. There are basically two types of sensitivity analysis; changes to the objective function and changes to right hand sides. In the farmer's problem changes are made to the profit margins of the two produce, wheat and potatoes, and to two of the constraints, the potato quota and the area of

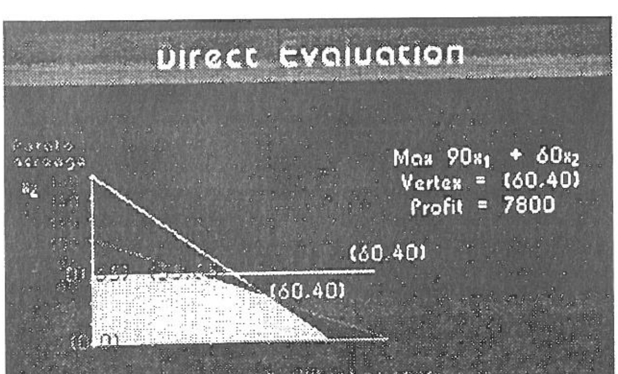

Fig. 5. Direct Evaluation of Vertices

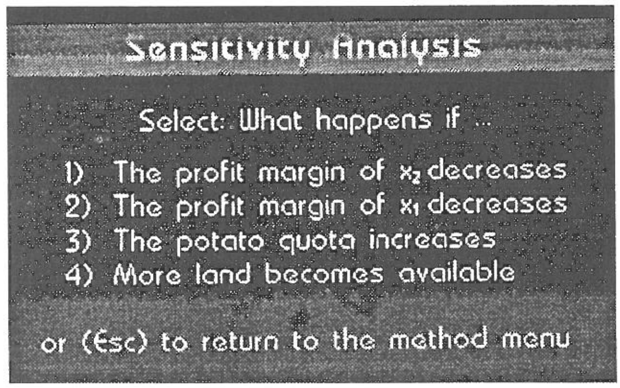

Fig. 6. Sensitivity Menu

land available. See Figure 6 for the sensitivity analysis menu.

When changes are made to the objective function a limit is reached after which the current basis (solution) is no longer optimal. This process can be illustrated by pivoting the objective function about the current solution. When the objective function becomes parallel to a constraint there are two alternative basic solutions. If the objective function continues to change it

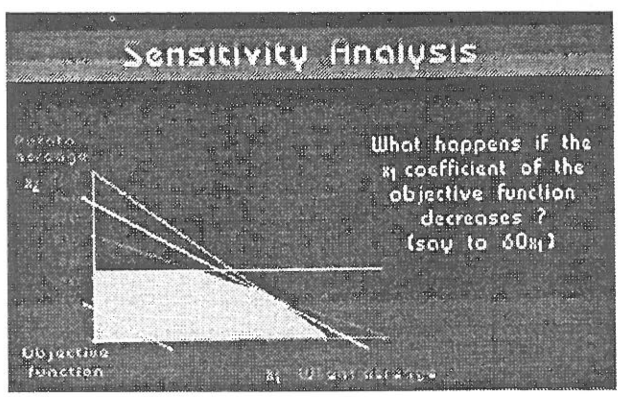

Fig. 7. Decreasing an Objective Function Coefficient

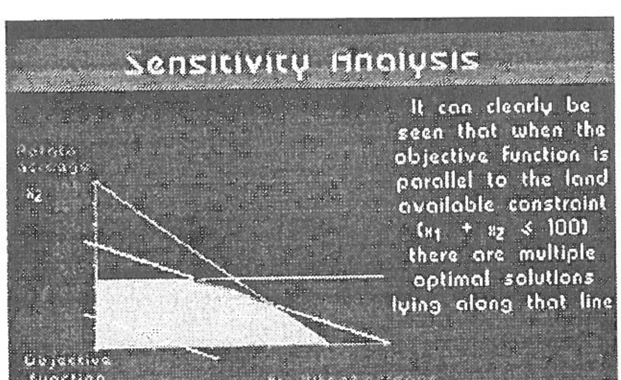

Fig. 8. Multiple Optimal Solutions

will pivot about the new basic solution. Figures 7 through 10 show some of the frames as the objective coefficient of one of the variables changes. Decreasing the coefficient of variable x_1 , wheat acreage, swings the objective function anti-clockwise until it becomes parallel to a constraint, the land constraint in this case. Further reduction changes the optimal solution.

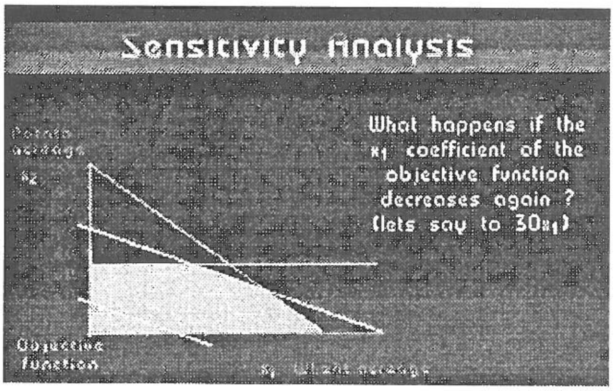

Fig. 9. Further Reduction in an Objective Function Coefficient

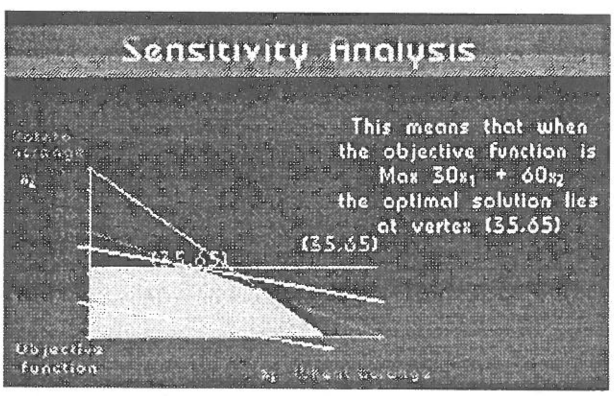

Fig. 10. A New Optimal Solution

Changes to right hand sides fall into two categories; non-binding and binding constraints. In the farmer's problem the potato quota is non-binding and the availability of land is binding. These changes are shown through simple animations. Increases in the potato quota will have no effect on the solution. The frames depicted in Figures 11 through 13 show the effect of increases to the potato quota constraint. Increasing the potato quota changes the shape of the feasible region but has no effect on the optimal solution. An increase in the area of land will change the amounts of potatoes and wheat grown, and, thus, change the value of the objective, as shown in Figures 14 and 15. The change in the objective is determined by the

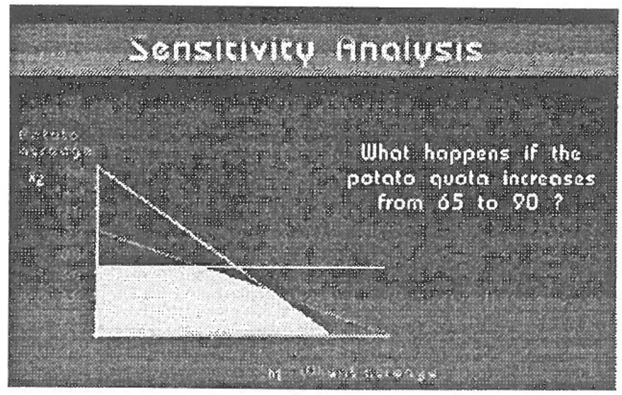

Fig. 11. Increasing a RHS of a Non-binding Constraint

shadow price of the constraint. The animation tries to convey the meaning of the shadow price in the context of this problem (Figure 15).

Attempts have been made to model three variable problems using animation techniques. This has been done with the software 3D Studio. This has proved to be much more difficult. The main problem is that in three dimensional problems some surfaces lie behind others and can be hidden. Experiments with different materials in 3D Studio have suggested the use of 'coloured

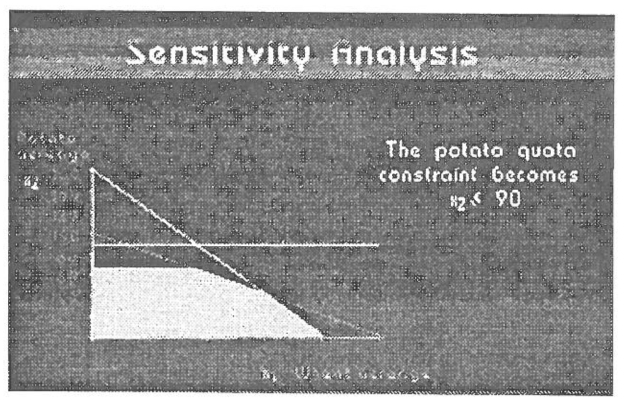

Fig. 12. An Extended Feasible Region

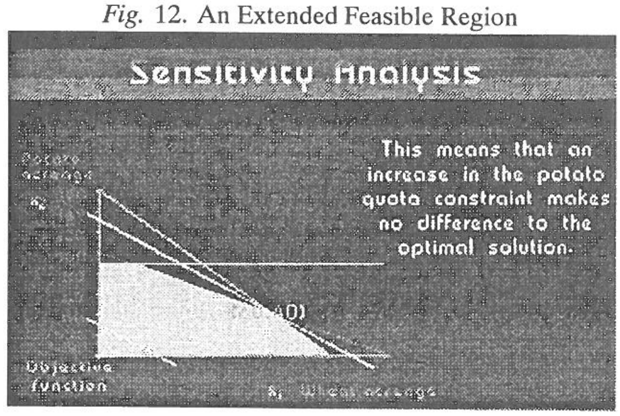

Fig. 13. The Optimal Solution is not Changed

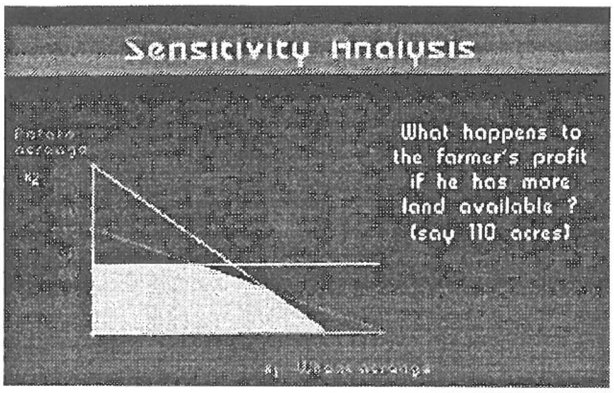

Fig. 14. Increasing the RHS of a Binding Constraint

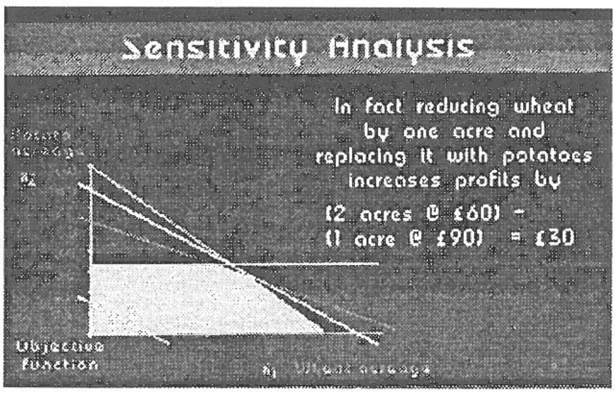

Fig. 15. A New Optimal Solution with Shadow Price

glass' materials can be effective. So far the animations created have not attempted to animate the construction of constraints. The constraints have been drawn and the animation displays the feasible region from various angles via a moving camera. Objective function planes are drawn for various iso profit values.

5. Learning Styles and Computer Assisted Learning

Cognitive style refers to the preferred mode of processing information. Learning strategies are cognitive styles applied by individuals in learning. "In a sense, styles are latent and strategies are manifest" (Allinson, 1992). A large number of learning styles have been identified and reported such as surface versus deep processing (Marton and Saljo, 1976) and holist versus serialist strategies (Pask and Scott, 1972) "but often, the styles differ more in name than in nature" (Clarke, 1993). The essential difference is the preference for a more structured presentation of information as against a preference for freedom to choose the direction in which the learning should proceed. One of many studies (Allinson, 1992) has described the former as a *high reproducing* approach to study and the latter as a *high meaning* approach. The Allinson study identified, using Entwistle's Approaches to Study Inventory (Entwistle, 1988), two groups of students at the extremes of the high meaning high reproducing continuum. The groups were introduced to two systems; Hitch-Hiker's Guide and Physiological Feedback in Humans. Both systems had several navigational aids; index, tour, map and hypertext links. The study showed that the high reproducing group showed a preference for a more linear and structured presentation and made greater use of the tour facility whereas the high meaning group made greater use of the hypertext links and the index facility. The learning outcomes as measured by subsequent exam ination results were similar for both groups.

The implication for education is presumably that teachers need to provide opportunities for students to learn in away which suits their preferred style of learning. If teachers adopt too extreme a method of teaching, perhaps reflecting their own learning style, one group of students will find the approach alien to their way of learning (Entwistle, 1981). In designing computer assisted learning systems we must attempt to cater for the wide variety of learning styles used by students.

6. Concluding Remarks

The system has only had limited exposure to students so far. It was made available for students during their revision of LP prior to the examinations in June 1993. The comments received from the students were favourable. The animation system described in this paper is certainly more suited to the high reproducing student, perhaps reflecting the author's own learning style. But, as mentioned earlier, this is not the only system that will be experienced by students. This animation system will be used alongside the Mentor LP module that is more suited to the high meaning students, and yet does not exclude the high reproducing students. Nevertheless, the animation system must be made more flexible to cater for all learning styles. The aim is to include the animations in a more flexible system, possibly incorporated into the Mentor LP module. XPRESSMP will also be used. As well as making the system more flexible future work will extend the range of examples, problems with $\geq$ constraints, minimisation problems etc., and animate the simplex method. Much more can be done given the time and a little (more) imagination. It is too early to say how the introduction of the animation and other systems will affect the method of teaching, however, I expect the number of lectures will decrease, the number computer laboratory sessions and open door sessions will increase, thereby maintaining or increasing the level of one-to-one contact with students even with relatively large class sizes.

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